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Renormalization of number density in nonequilibrium quantum-field theory and absence of pinch singularities

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Abstract

Through introducing a notion of renormalization of particle-number density, a simple perturbation scheme of nonequilibrium quantum-field theory is proposed. In terms of the renormalized particle-distribution functions, which characterize the system, the structure of the scheme (and then also the structure of amplitudes and reaction rates) are the same as in the equilibrium thermal field theory. Then, as an obvious consequence, the amplitudes and reaction rates computed in this scheme are free from pinch singularities due to multiple products of δ-functions, which inevitably present in traditional perturbation scheme.

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Since late fifties, efforts have been made to incorporate quantum field theory with nonequilibrium statistical mechanics [1, 2, 3]. In connection with the description of quark-gluon plasma, which is expected to be produced in heavy-ion collisions and to have existed in the early universe, much work has recently been devoted to this issue (see, e.g., [4, 5, 6, 7, 8, 9]).

The standard framework of nonequilibrium theory is formulated by employing the closed-time path in a complex-time plane. A few years ago, Altherr and Deibert [7] have pointed out that, when calculating amplitudes or physical quantities in perturbative closed-time-path formalism, the "pathological terms" [3, 4] necessarily appear. More precisely, such terms appear in self-energy-parts-inserted two-point functions in association with pinch singularities coming from multiple products of δ-functions. These terms do not cancel each other unless the particle distributions are those for a system in thermal and chemical equilibrium. Since then it is shown that classes of such pathological terms can be resummed [8] (see also [2]). An application of this result to the rate of hard-photon production from a nonequilibrium quark-gluon plasma system is made in [9].

In this letter, introducing a notion of renormalization of number density, we propose a simple perturbation scheme of quantum field theory for computing the rates of reactions taking place in a quasiuniform system near equilibrium or nonequilibrium quasistationary system. The scheme is the same in structure as the equilibrium thermal field theory (ETFT), provided that the equilibrium number density $n(p_0)$ in ETFT is replaced by the renormalized nonequilibrium number density $N(P)$ that characterizes the system under consideration. Then, in contrast to the canonical perturbation scheme, no "pathological terms" mentioned above appear in amplitudes and reaction rates evaluated within the scheme.

We employ the closed-time-path formalism [2, 3, 4]. In this formalism, propagators, vertices and self-energy parts enjoy 2×2 matrix structure. We use $\hat{B}$ to denote the 2×2 matrix whose $(i\,j)$ component is B_{ij}. A vertex matrix $\hat{V}$ has simple structure, $V_{12} = V_{21} = 0$ and $V_{11} = -V_{22}$, with V_{11} the vertex factor in vacuum theory. Let $\hat{\Delta}(x_1, x_2)$ and $\hat{\Sigma}(x_1, x_2)$ be, in respective order, the bare propagator and the self-energy part in a configuration space. Following [2], we assume that reactions taking place in a system under consideration are "described" by the relative or microscopic

coordinates $x = x_1 - x_2$. Then, we make Fourier transforms with respect to x^μ, $\hat{\Delta}(P, X)$ and $\hat{\Sigma}(P, X)$, where $X = (x_1 + x_2)/2$ is the center-of-mass or macroscopic coordinates. It is assumed [2] that the dependence of $\hat{\Delta}(P, X)$ and $\hat{\Sigma}(P, X)$ upon X^μ is weak. More precisely, over the macroscopic space-time region of the system, where a microscopic or elementary reaction takes place, X^μ-dependence of $\hat{\Delta}(P, X)$ and $\hat{\Sigma}(P, X)$ may be ignored. Then, in calculating the reaction rate, we can use $\hat{\Delta}(P, X)$ and $\hat{\Sigma}(P, X)$ with fixed X^μ. This is the first stage of the theoretical analysis. Microscopic reactions cause changes in the number densities of (quasi)particles, through which the density matrix changes with macroscopic space-time X^μ. Dealing with this is the subject of the second stage, where (weak) X^μ-dependence of $\hat{\Delta}(P, X)$ and $\hat{\Sigma}(P, X)$ are explicitly taken into account. In this letter, as in [7, 8, 9], we concentrate our concern on the first stage and drop the argument X throughout.

The matrix elements of $\hat{\Delta}(P)$ and $\hat{\Sigma}(P)$ enjoy various properties [2]. Among those we shall use $Re\Sigma_{12} = Re\Sigma_{21} = 0$, $\Sigma_{22} = -\Sigma_{11}^*$, $\sum_{i,\,j=1}^{2}(-)^{i+j}\Delta_{i\,j}(P) = 0$ and $\sum_{i,\,j=1}^{2}\Sigma_{ij}(P) = 0$.

For simplicity of presentation, we take a massless, self-interacting, complex-scalar field theory with a conserved charge. $\hat{\Delta}(P)$ may be written as

$$\hat{\Delta}(P) = \hat{M}(P)\,\hat{\Delta}_F(P)\,\hat{M}(P)\,, \tag{1}$$

where[†]

$$\hat{\Delta}_F(P) = \mathrm{diag}\,(\Delta_F(P),\, -\Delta_F^*(P)) \qquad \Big(\Delta_F(P) = 1/(P^2 + i\epsilon)\Big)\,, \tag{2}$$

$$\hat{M}(P) = \begin{pmatrix} \sqrt{1+n(P)} & \frac{\theta(-p_0)+n(P)}{\sqrt{1+n(P)}} \\ \frac{\theta(p_0)+n(P)}{\sqrt{1+n(P)}} & \sqrt{1+n(P)} \end{pmatrix}. \tag{3}$$

Here $n(P) = n(p_0,\, p)$ with $p = |\mathbf{p}|$ is the number-density function that characterizes the ensemble of the systems. In the case where the "local" temperature $T = \beta^{-1}$ and the chemical potential μ, being conjugate to the charge, are defined, $n(P) = 1/[e^{\beta(|p_0|-\epsilon(p_0)\mu)} - 1]$ with β and μ the functions of macroscopic space-time coordinates.

Here we recall that, in ETFT, the self-energy-part matrix $\hat{\Sigma}(P)$ takes [3] the form

$$\hat{\Sigma}(P) = \hat{M}^{-1}(P)\,\hat{\Sigma}_F(P)\,\hat{M}^{-1}(P) \qquad \hat{\Sigma}_F(P) = \mathrm{diag}\,(\Sigma_F(P) - \Sigma_F^*(P))\,. \tag{4}$$

[†] In calculating an amplitude in ETFT, a common practice [3] is to keep ϵ finite throughout and at the end of calculation the limit $\epsilon \to 0^+$ is taken. We follow this procedure.

One can easily see from Eqs. (1), (2) and (4) that an l (≥ 1) self-energy-parts-inserted two-point function $\hat{G}_l(P) \equiv \hat{\Delta}(P)[\hat{\Sigma}(P)\,\hat{\Delta}(P)]^l$ does not include functions $(P^2+i\epsilon)^{-k}(P^2-i\epsilon)^{-(l+1-k)}$ ($1 \leq k \leq l$) but includes functions $(P^2 \pm i\epsilon)^{-(l+1)}$. In the limit $\epsilon \to 0^+$, the former functions develop pinch singularity while the latter functions turn out to the well-defined distributions, and thus $\hat{G}_l(P)$ is free from pinch singularity. In the present nonequilibrium case, Eq. (4) does not hold and $\hat{G}_l(P)$ includes "pinch-singular" functions $(P^2+i\epsilon)^{-k}(P^2-i\epsilon)^{-(l+1-k)}$. This is essentially what has been observed in [7].

Let us construct the renormalization theory by introducing a renormalized or "physical" number density $N(P)$,

$$N(P) = n(P) + \delta n(P)\,, \tag{5}$$

with which the bare (renormalized) propagator reads (cf. Eqs. (1) - (3))

$$\begin{aligned} \hat{\Delta}^{(r)}(P) &= \hat{\Delta}(P)\Big|_{n(P)\,\to\,N(P)}\,, \\ \hat{M}^{(r)}(P) &= \hat{M}(P)\Big|_{n(P)\,\to\,N(P)}\,. \end{aligned} \tag{6}$$

In order to compensate the difference between the renormalized $\hat{\Delta}^{(r)}$ and the original "bare" $\hat{\Delta}$, $\delta\hat{\Delta}(P) \equiv \hat{\Delta}^{(r)}(P) - \hat{\Delta}(P)$, we should introduce the counter term in the interaction Lagrangian,

$$L_c = -\frac{1}{2}\int \prod_{\ell=1}^{4}\left(d^4x_\ell\right) \sum_{i,\,j=1}^{2} \phi_i^*(x_1)\left[\hat{\Delta}^{-1}(x_1,\,x_2)\,\delta\hat{\Delta}(x_2,x_3)\,\hat{\Delta}^{-1}(x_3,\,x_4)\right]_{ij}\phi_j(x_4)\,, \tag{7}$$

where ϕ_j and ϕ_j^* ($j=1,2$) stand for the type-j fields and

$$\begin{aligned} \hat{\Delta}^{-1}(x,y) &\equiv \int \frac{d^4P}{(2\pi)^4}\,e^{-iP\cdot(x-y)}\,\hat{\Delta}^{-1}(P) \\ \delta\hat{\Delta}(x,y) &\equiv \int \frac{d^4P}{(2\pi)^4}\,e^{-iP\cdot(x-y)}\,\delta\hat{\Delta}(P) \\ &= \int \frac{d^4P}{(2\pi)^4}\,e^{-iP\cdot(x-y)}\left[-2\pi i\,\delta n(P)\,\delta_\epsilon(P^2)\,\hat{A}\right]\,. \end{aligned} \tag{8}$$

Here $A_{ij} = 1$ ($i,\,j = 1,\,2$) for all i and j and[†] $\delta_\epsilon(P^2) \equiv \epsilon/[\pi\{(P^2)^2+\epsilon^2\}]$.

The perturbative calculation goes with propagator $\hat{\Delta}^{(r)}(P)$ in Eq. (6) and "vertices" coming from $L_{int}+L_c$, where L_{int} is the original interaction Lagrangian. Conforming to the ultra-violet renormalization theory, one can say that $n(P)$ is the bare

number-density function and $N(P)$ is the renormalized or physical number-density function. The latter describes the physical system under consideration. The self-energy part $\hat{\Sigma}^{(r)}(P)$ obtained from the above perturbation scheme takes the form[‡]

$$\hat{\Sigma}^{(r)}(P) = \hat{\Sigma}(P) - \hat{\Delta}^{-1}(P)\,\delta\hat{\Delta}(P)\,\hat{\Delta}^{-1}(P)\,. \tag{9}$$

We are now in a position to set up the renormalization condition: *The self-energy part $\hat{\Sigma}^{(r)}(P)$ is the same in structure as Eq. (4) in ETFT*, i.e.,

$$\hat{\Sigma}^{(r)}(P) = [\hat{M}^{(r)}(P)]^{-1}\,\hat{\Sigma}_F^{(r)}(P)\,[\hat{M}^{(r)}(P)]^{-1} \tag{10}$$

where

$$\hat{\Sigma}_F^{(r)}(P) = \mathrm{diag}\left(\Sigma_F^{(r)}(P),\, -(\Sigma_F^{(r)}(P))^*\right)\,. \tag{11}$$

The second term on the right-hand side of Eq. (9) does not contribute to $(\Sigma_F^{(r)})_{11}$ and $(\Sigma_F^{(r)})_{22}$ and the condition $[\{\Sigma_F^{(r)}(P)\}_{22}]^* = -\{\Sigma_F^{(r)}(P)\}_{11}$ is automatically met;

$$\Sigma_F^{(r)}(P) = \Sigma_{11}(P) + \theta(p_0)\,\Sigma_{12}(P) + \theta(-p_0)\,\Sigma_{21}(P)\,. \tag{12}$$

From Eq. (6) with Eq. (3) and Eqs. (8) and (9), we see that the condition $\{\Sigma_F^{(r)}(P)\}_{12} = \{\Sigma_F^{(r)}(P)\}_{21} = 0$ yields

$$\begin{aligned}\delta n(P)\,\delta_\epsilon(P^2) &= -\frac{i}{2\pi}\,\frac{1}{P^2 - i\epsilon}\,\frac{1}{P^2 + i\epsilon}\,[\theta(p_0)\,\{(1+N(P))\,\Sigma_{12}(P) - N(P)\,\Sigma_{21}(P)\} \\ &\quad +\theta(-p_0)\,\{(1+N(P))\Sigma_{21}(P) - N(P))\,\Sigma_{12}(P)\}]\,.\end{aligned} \tag{13}$$

Now let us introduce

$$\begin{aligned}\Gamma_p^{(net)}(P) &\equiv \theta(p_0)[\{1+N(P)\}\Gamma_p(P) - N(P)\Gamma_d(P)] \\ &\quad +\theta(-p_0)[\{1+N(P)\}\overline{\Gamma}_p(P) - N(P)\overline{\Gamma}_d(P)]\,,\end{aligned} \tag{14}$$

where

$$\begin{aligned}\Gamma_p(P) &= \overline{\Gamma}_d(P) = -\frac{i}{2p}\,\Sigma_{12}(P)\,, \\ \Gamma_d(P) &= \overline{\Gamma}_p(P) = -\frac{i}{2p}\,\Sigma_{21}(P)\,.\end{aligned} \tag{15}$$

[‡] Suppose that we are analyzing $\hat{\Sigma}^{(r)}$ in the nth order of perturbation series. The second term on the right-hand side of Eq. (9) comes from L_c with the nth order counter term, being proportional to $\delta n^{(n)}(P)$. The nth order $\hat{\Sigma}(P)$ in Eq. (9) involves $\delta n^{(j)}(P)$ with $1 \le j \le n-1$.

On the mass shell $p_0 = p$ ($p_0 = -p$), $\Gamma_p^{(net)}(P)$ is the net production rate of a particle (antiparticle) with momentum $\mathbf{p}$ ($-\mathbf{p}$) [2]. Then $\Gamma_p^{(net)}(P)$ in Eq. (14) is an off-shell extension of the net production rate. Substitution of Eq. (14) with Eq. (15) into Eq. (13) yields

$$\delta n(P)\, \delta_\epsilon(P^2) = \frac{p}{\pi}\, \frac{1}{(P^2)^2 + \epsilon^2}\, \Gamma_p^{(net)}(P)\,. \tag{16}$$

It is to be noted in passing that, in ETFT, $\Gamma_p^{(net)}(P)$ vanishes (see, e.g., [2]).

In the limit $\epsilon \to 0^+$, $\delta n(P)$ in Eq. (16) is singular due to the pinch singularity at $|p_0| = p$. Equation (16) may be "solved" for $\delta n(P)$ in the form,

$$\begin{aligned} \delta n(P) &= 2(p^2 + \tilde{\epsilon}^2)\, \Gamma_p^{(net)}(P) \int_{-\infty}^{+\infty} \frac{dk_0}{2\pi}\, \Delta_F^*(k_0,\, p)\, \Delta_F(k_0,\, p) &(17)\\ &= \frac{p}{\epsilon}\, \Gamma_p^{(net)}(P)\,, &(18) \end{aligned}$$

where $\tilde{\epsilon} \simeq \epsilon/2p$.

It is obvious from the above construction that the renormalization theory is obtained from the ETFT simply by substituting $N(P)$ for the equilibrium distribution function $n_B(p_0)$ ($= 1/[e^{\beta(|p_0| - \epsilon(p_0)\mu)} - 1]$). Then, the fact in ETFT that a self-energy-parts-inserted propagator is free from pinch singularities (cf. above after Eq. (4)) is transmitted as it is to the case of renormalization theory. In fact, from Eq. (6) with Eqs. (1) and (2), and Eqs. (10) and (11), we see that $\hat{\Delta}^{(r)}(P)[\hat{\Sigma}^{(r)}(P)\, \hat{\Delta}^{(r)}(P)]^l$ ($l \geq 1$) includes only well-defined function $(P^2 \pm i\epsilon)^{-(l+1)}$.

We now demonstrate that, as far as the *singular pieces* are concerned, $n(P)$ on the mass shell $p_0 = p$ [$p_0 = -p$] coincides with the Heisenberg particle [antiparticle] number density $n_a^H(p)$ [$n_b^H(p)$]. $n_a^H(p)$ reads, with obvious notation,

$$\begin{aligned} \langle a^\dagger(\mathbf{p},\, t)\, a(\mathbf{p}',\, t)\rangle &\equiv \delta(\mathbf{p} - \mathbf{p}')\, n_a^H(p) \\ &= \frac{1}{2p} \int \frac{d^3x\, d^3y}{(2\pi)^3}\, e^{-i(P\cdot x - P'\cdot y)}\, \frac{\overleftrightarrow{\partial}}{\partial x_0}\, \frac{\overleftrightarrow{\partial}}{\partial y_0}\, \langle \phi^*(x)\, \phi(y)\rangle \Big|_{x_0\, =\, y_0\, =\, t}\,, \end{aligned} \tag{19}$$

where $P = (p, \mathbf{p})$ and $P' = (p', \mathbf{p}')$. For the case of antiparticle number density, we have Eq. (19) with $\phi^* \leftrightarrow \phi$. Note that $\langle \phi^*(x)\, \phi(y)\rangle$ [$\langle \phi(x)\, \phi^*(y)\rangle$] is the (12) [(21)] component [2, 3, 4] of the full propagator $iG_{12}(y - x)$ [$iG_{21}(x - y)$], we obtain

$$n_{a/b}^H(p) = \frac{1}{2p} \int_{-\infty}^{+\infty} \frac{dk_0}{2\pi}\, (k_0 \pm p)^2\, iG_{12/21}(k_0,\, p)\,. \tag{20}$$

We compute $n^H_{a/b}(p)$ up to the contribution from one self-energy-part-inserted $G_{12/21}(k_0,\, p)$. We first employ the perturbation scheme in the renormalization theory defined above. As mentioned above, computation goes just as in the case of ETFT. The resultant $n^H_{a/b}(p)$ reads

$$n^H_{a/b}(p) = N(\pm p,\, p) + \delta N(\pm p,\, p)\,. \tag{21}$$

Here N comes from the lowest-order contribution to $G_{12/21}$ and δN comes from the one self-energy-part-inserted $G_{12/21}$. As in the case of ETFT, δN is a well-defined at most finite functional of N.

Now we turn to compute $n^H_{a/b}(p)$ in *traditional scheme*, without introducing the renormalization counter term. The lowest-order contribution to $G_{12/21}$ yields $n^H_{a/b}(p)$ $= n(\pm p,\, p)$, where n is the "starting" number density function in Eq. (3). Then, we write

$$n^H_{a/b}(p) = n(\pm p,\, p) + \delta n^H_{a/b}(p)\,. \tag{22}$$

The contribution from one self-energy-part-inserted $G_{12/21}$ to $\delta n^H_{a/b}(p)$ reads,

$$\begin{aligned} \delta n^H_{a/b}(p) \;\; = \;\; & \int_{-\infty}^{+\infty} \frac{dk_0}{2\pi}\,(k_0 \pm p)^2 \left[\Delta_F^*\,\Delta_F\,\tilde{\Gamma}^{(net)}_p(k_0,\, p)\right. \\ & \left. -(\theta_\mp + n)\left\{(\Delta_F)^2\tilde{\Sigma}_F - (\Delta_F^*)^2\tilde{\Sigma}_F^*\right\}\right]\,, \end{aligned} \tag{23}$$

where $\tilde{\Sigma}_F = \tilde{\Sigma}_{11} + \theta_+\tilde{\Sigma}_{12} + \theta_-\tilde{\Sigma}_{21}$, $\theta_\pm \equiv \theta(\pm k_0)$, $n = n(k_0,\, p)$, $\Delta_F = \Delta_F(k_0,\, p)$ and $\tilde{\Sigma}_F = \tilde{\Sigma}_F(k_0,\, p)$. $\tilde{\Gamma}^{(net)}_p(k_0, p)$ in Eq. (23) is the traditional-scheme counterpart of $\Gamma^{(net)}_p(P)$ in Eq. (14) with Eq. (15). In the limit $\epsilon \to 0^+$, the term with $\Delta_F^*\,\Delta_F$ in Eq. (23) diverges due to pinch singularity at $|k_0| = p$. On the other hand, the contributions from $(\Delta_F)^2$ and $(\Delta_F^*)^2$, being well-defined functions, are at most finite. We deal only with the singular contribution coming from $|k_0| = p$,

$$\delta n^H_{a/b}(p) \simeq 2p^2\,\tilde{\Gamma}^{(net)}_p(\pm p,\, p)\int_{-\infty}^{+\infty}\frac{dk_0}{2\pi}\Delta_F^*(k_0,\, p)\,\Delta_F(k_0,\, p)\,, \tag{24}$$

where '$\simeq$' is used to denote an approximation that is valid for keeping the singular contribution. In obtaining Eq. (24), use has been made of the fact that $\Delta_F^*\,\Delta_F$ is even function of k_0. $\delta n(P)$ on the mass shell $p_0 = \pm p$, Eq. (17), and the singular piece of $\delta n^H_{a/b}(p)$ in Eq. (24) is the same in form. It should be noted, however, that $\tilde{\Gamma}^{(net)}_p \neq \Gamma^{(net)}_p$. We rewrite $\tilde{\Gamma}^{(net)}_p$ in terms of N by using $n(\pm p,\, p) = N(\pm p,\, p) + \{n(\pm p,\, p) -$

$N(\pm p,\, p)\}$. The philosophy of renormalization tells us that the difference $n - N$ only affects the higher order contribution to $n^H_{a/b}(p)$. Thus, to the accuracy of one self-energy-part insertion, we have $\delta n^H_{a/b}(p) \simeq \delta n(\pm p,\, p)$. Then, from Eqs. (21) and (22), we find

$$\begin{aligned} n^H_{a/b}(p) \simeq N(\pm p,\, p) &\simeq n(\pm p, p) + \delta n^H_{a/b}(p) \\ &\simeq n(\pm p,\, p) + \delta n(\pm p,\, p)\,, \end{aligned} \tag{25}$$

where use has been made of the fact that δN, Eq. (21), is not singular. The relation (25) is in accord with Eq. (5) on the mass shell $p_0 = \pm p$.

Thus, we have learned that, as far as the singular pieces are concerned, the renormalized number density $N(P)$ (Eq. (5)) on the mass shell $p_0 = \pm p$ coincides with the Heisenberg number density $n^H_{a/b}(p)$. Then we may regard $N(P)$ as an off-shell extension of the Heisenberg number density.

In passing it is worth making a comment. The first computation leading to Eq. (21) is based on the theory, in which the free Lagrangian is written in terms of physical or renormalized number density $N(P)$. On the other hand, the second computation is based on the theory, in which the free Lagrangian is written in terms of original bare number density $n(P)$. As in the ultra-violet renormalization theory in quantum field theories, both approaches are equivalent. In the first approach, the renormalization is "done" at the beginning by introducing the counter Lagrangian L_c. While the second approach starts with the bare Lagrangian and the renormalization is "done" at the end.

Now let us turn to analyze the physical meaning of the *singular part* of $\delta n^H_{a/b}(p)$, Eq. (24). Introducing the Fourier transform of $\Delta_F(k_0,\, p)$, Eq. (2),

$$\begin{aligned} \Delta_F(t,\, p) &\equiv \int_{-\infty}^{+\infty} \frac{dk_0}{2\pi}\, e^{-ik_0 t} \Delta_F(k_0,\, p) \\ &= -\frac{i}{2p}\left[\theta(t) e^{-i(k - i\tilde{\epsilon})t} + \theta(-t) e^{i(k - i\tilde{\epsilon})t}\right], \end{aligned} \tag{26}$$

we obtain

$$\int_{-\infty}^{+\infty} \frac{dk_0}{2\pi}\, \Delta^*_F(k_0,\, p)\, \Delta_F(k_0,\, p) = \int_{-\infty}^{+\infty} dt\, \Delta^*_F(-t,\, p)\, \Delta_F(t,\, p)\,. \tag{27}$$

Here let us recall that, in standard Lippmann-Schwinger formalism, a singular function $2\pi[\delta(E_f - E_i)]^2$ appears in a transition probability of some reaction, where E_i

(E_f) is the energy of the initial (final) state of the reaction. We follow the standard argument for interpreting this function as $(t_f - t_i)\delta(E_f - E_i)$, where $t_f - t_i$ is the time interval during which the interaction acts. Applying this to the present case, we have

$$\begin{aligned} \int_{t_i}^{t_f} dt\, \Delta_F^*(-t,\, p)\, \Delta_F(t,\, p) &= \frac{1}{4p^2}\left[\frac{1 - e^{2\tilde{\epsilon}t_i}}{2\tilde{\epsilon}} - \frac{e^{-2\tilde{\epsilon}t_f} - 1}{2\tilde{\epsilon}}\right] \\ &\simeq \frac{1}{4p^2}\,(t_f - t_i)\,. \end{aligned} \tag{28}$$

Substituting this into Eq. (27) and then into Eq. (24), we obtain

$$\delta n_{a/b}^H(p) \simeq \frac{t_f - t_i}{2}\,\Gamma_p^{(net)}(\pm p,\, p)\,. \tag{29}$$

It is to be noted in passing that if we take the limit $t_i \to -\infty$ and $t_f \to +\infty$ in Eq. (28), we have $1/4p^2\tilde{\epsilon} \simeq 1/2p\epsilon$. Substitution of this into Eq. (24) reproduces Eq. (18) on the mass shell $p_0 = \pm p$.

It is interesting to note that Eq. (29) is *half* of the net production probability during the time interval $t_f - t_i$ of the reaction. What the result (29) tells us is the following. Since $\Gamma_p^{(net)} \neq 0$, the number density $\tilde{n}_{a/b}(p,\, t)$ changes with microscopic time t. At the initial time t_i $(\sim -\infty)$ of some reaction $\tilde{n}_{a/b}(p,\, t_i) = n(\pm p,\, p)$, Eq. (3), and at the final time t_f $(\sim +\infty)$ $\tilde{n}_{a/b}(p, t_f) = \tilde{n}_{a/b}(p,\, t_i) + (t_f - t_i)\Gamma_p^{(net)}(\pm p,\, p) \simeq n(\pm p,\, p) + 2\delta n_{a/b}(p)$. Then, $\delta n_{a/b}^H(p)$ in Eq. (29) is $\delta n_{a/b}^H(p) = \tilde{n}_{a/b}(p, (t_f + t_i)/2) - \tilde{n}_{a/b}(p, t_i)$, i.e., $N(\pm p, p) \simeq n(\pm p, p) + \delta n_{a/b}^H(p)$, Eq. (25), is the number density $\tilde{n}_{a/b}(p, t)$ at the middle or average time of the initial and final times of the reaction, $t = (t_f + t_i)/2$.

Here we briefly mention how to treat the gauge field and massless fermion field. In the case of gauge field like gluon in QCD, we employ the Coulomb gauge or the Landshoff-Rebhan's variant [10] of the covariant gauge. The $n(P)$-dependent part of $\hat{\Delta}(P)$ and $\hat{\Sigma}(P)$ are decomposed into two sectors, the electric sector (e) and the magnetic sector (m). For each sector τ $(= e, m)$, we introduce the Bogoliubov matrix, which is given by Eq. (3) with $n(P) \to n^{(\tau)}(P)$. For massless fermion field, we decompose $\hat{\Delta}(P)$ and $\hat{\Sigma}(P)$ into two sectors, $\hat{\Delta}(P) = \sum_{\tau=\pm} \mathcal{P}_\tau \hat{\Delta}^{(\tau)}(P)$ and $\hat{\Sigma}(P) = \sum_{\tau=\pm} \mathcal{P}_\tau \hat{\Sigma}^{(\tau)}(P)$, where $\mathcal{P}_\tau \equiv (\gamma_0 - \tau\hat{\mathbf{p}}\cdot\vec{\gamma})/2$. In particular, $\hat{\Delta}_F(P) = \sum_{\tau=\pm} \mathcal{P}_\tau \hat{\Delta}_F^{(\tau)}(P)$ with $\hat{\Delta}_F^{(\tau)}(P) = \mathrm{diag}[1/\{p_0(1 + i\epsilon) - \tau p\}, -1/\{p_0(1 - i\epsilon) - \tau p\}]$ (cf. Eq. (2)). For each sector τ $(= \pm)$, we introduce the Bogoliubov matrix, Eq. (3), with $n(P) \to -n^{(\tau)}(P)$. With this preliminaries, the analysis goes as in the case of complex-scalar field.

Finally we mention two related work. The first one is the so-called nonequilibrium thermo field dynamics [5]. This is a "single-time" formalism, without distinction between the microscopic- and macroscopic-times. The time representation, rather than the p_0 representation, is employed and through carrying out renormalization of the number density a nonrelativistic Schrödinger field theory is analyzed. Since the time representation is used, the structure of the (Schrödinger) propagators in a p_0 space may not be seen directly. The second one [6] starts with performing the approximate resummation of the absorptive parts of the self-energy parts in relativistic scalar field theories. Then, the perturbation scheme is so constructed that the renormalized free Lagrangian $L_0^{(r)}$ yields the approximately resummed propagator. The counter Lagrangian (the difference between the original free Lagrangian and $L_0^{(r)}$) is local. By contrast, the counter Lagrangian (7) in our renormalization scheme is nonlocal as in the hard-thermal-loop resummation scheme [4, 11] in ETFT.

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